in the A & V report. It is clear from the figure that there is a distinct difference between the corrected values taken directly from Fig. 13 in Ref. 1 and the corrected values computed from Eqs. (1) or (2) [also Eq. (67) in Ref. 1] herein. Note that the difference between the two sets of corrected values increases as the (d/h) ratio increases. These two sets of numbers should be the same since the values taken from the A & V report were determined from Eq. (1) in simplified form [which is Eq. (2)]. I would also like to point out that the abscissa on Fig. 13 of Ref. 1 is incorrect for circular cylinders. The lower abscissa on our Fig. 1 is taken from Fage's data; the upper abscissa on our Fig. 1 is that given by Allen and Vincenti. The lower abscissa on this figure is the correct one to use.

Based on Fage's experimental data, the plotted values of Allen and Vincenti for the freefield drag coefficient C_d are very close to the expected value of 1.2. My calculations with Eq. (2) [or, equivalently, Eq. (67) in Ref. 1] and Fage's data have shown that the freefield drag-coefficient values of C_d are increasingly less than 1.2 as the relative spacing ratio (d/h) increases. Note that these deviations from the expected value of 1.2 become significant for (d/h) spacings greater than about 0.1.

I feel that the reason for the deviation of the A & V calculations from the results obtained from Eq. (2) herein has been found. In what was intended to be a simple check of computational procedure, I recalculated the A & V results, obtaining their values for the corrected coefficients. However, after completion of these calculations, it was noticed that, in using Eq. (2), a mistake was made on one of the coefficients. Instead of using 2.472, I had used 0.2472 and had obtained the A & V values. Upon repeating the calculations with the proper tabulated value of 2.472, I found the corrected values to be those shown in Fig. 1 and labeled "Correction from Eq. (2)." Therefore, it seems that the A & V method produced good agreement with the data because of a simple decimal oversight in the calculational procedure.

Summary

These observations indicate that the A & V blockage corrections for drag coefficients on a circular cylinder in a wind tunnel should not be used for spacing ratios greater than 0.1. It would appear that the potential-flow model as posed by Allen and Vincenti does not accurately represent the real-flow situation for a circular cylinder when the spacing ratio becomes too large. However, I did find that the A & V method gives good agreement with the data for all available spacing ratios when the following equation is used instead of Eq. (2):

$$C_d = C_d'\{1 - \tfrac{1}{4}(d/h)^2 - (C_d'/2)(d/h)\} \qquad (3)$$

I suggest that Eq. (3) be used as a replacement for the A & V method for computation of blockage-corrected drag coefficients. I make this suggestion with no intent to cast doubt on the theoretical analysis set forth by Allen and Vincenti, but simply to have available a calculational procedure for blockage corrections which produces results in agreement with free-field values.

As an alternate method for circular-cylinder blockage corrections with large spacing ratios, I suggest that the method of Fage[5] as outlined in Durand[6] be used.

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Influence of the Injection Conditions on the Ignition of Methane and Hydrogen in a Hot Mach 2 Air Stream

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In a preceding investigation the initiation and propagation of combustion was studied for transverse fuel injection into supersonic air streams heated by a 350 kW plasma burner.[1] Gaseous hydrogen and methane were injected through a cylindrical nozzle of 1.56 mm ϕ at an angle of 90° into free, parallel air streams with Mach numbers between 2 and 3 and static temperatures ranging from $\approx$600° to $\approx$2000°C. In the following, further experiments concerning the influence of the injection angle and of an adjacent wall on the ignition temperature are described.

The experiments were conducted with the same set-up as used in the previous investigation. Figure 1 shows the geometrical arrangement for the injection of the fuel gas through an inclined cylindrical nozzle of d_s = 1.5 mm diam and through a vertical cylindrical hole of the same diameter in a flat water-cooled copper plate, adjusted tangentially to the air flow. In the latter case, ambient air could be introduced through a slit of s = 0.4 mm width into the wake behind the fuel gas jet. As in the previous investigation, the "ignition temperature" t_z, that is the minimum static temperature of the undisturbed air stream causing ignition, was determined as a function of the fuel pressure ratio p_{os}/p_k (p_{os} = stagnation pressure of the fuel, p_k = static pressure in the undisturbed air stream, equal to atmospheric pressure). The stagnation temperature of the fuel was always 20°C.

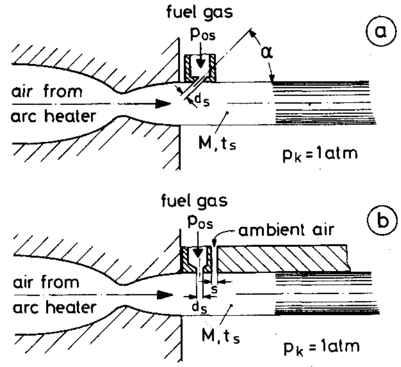

Fig. 1 a) Schematic of the fuel injection through an inclined nozzle and b) a boring in a plate.

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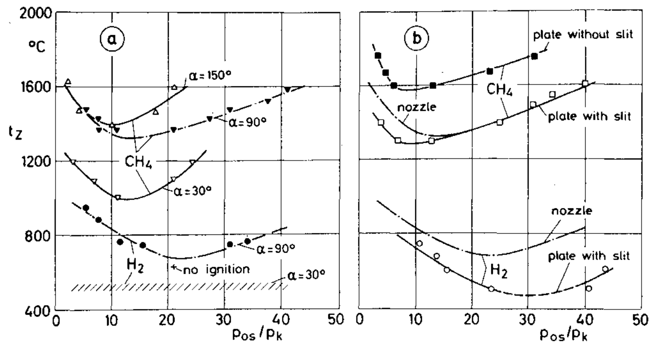

Fig. 2 Ignition temperature of a Mach 2 air stream for fuel injection through nozzles and b) plates.

Fuel Injection through Single Nozzles

Figure 2a shows typical results for fuel gas injection through single nozzles with different inclination angles into an air stream of $M = 2$. The curves for $\alpha = 90°$ were taken from the previous work.[1] Whereas the initial decrease of the ignition temperature with increasing fuel pressure ratio can be explained by the steepening of the bow shock in front of the injected fuel gas, the increase of the ignition temperature beyond the minimum is most probably due to the fuel jet penetrating through the relatively thin air jet of $\approx$10 mm diam.

When the fuel jet is directed *upstream* ($\alpha = 30°$) the ignition temperature decreases markedly compared with vertical injection, the difference between the minimum ignition temperatures for $\alpha = 90°$ and $\alpha = 30°$ amounting to $\approx$400°C for methane. With hydrogen injected at an angle $\alpha = 30°$, stable combustion was observed at static air temperatures as low as 500°C for all pressure ratios of the fuel. It was not possible to determine the corresponding minimum ignition temperature as a function of the pressure ratio because, with the present set-up of the arc heater, the static temperature of the Mach 2 air stream could not be reduced below 500°C. On the other hand, the ignition temperature is shifted to higher values, when the fuel gas nozzle is directed *downstream*, as is shown by the curve for methane at $\alpha = 150°$. The influence of the injection angle can easily be explained by the fact that the strength of the bow shock in front of the fuel gas and, therefore, the increase in temperature and density are smaller for downstream than for upstream injection. It is to be expected that for injection angles approaching $\alpha = 180°$ the ignition temperature rises towards the value for parallel fuel admixture.

Fuel Injection through the Plate

Since it was an open question, whether the results obtained for fuel injection through single nozzles into free supersonic air streams can be applied to channel flows, the influence of adjacent walls was investigated qualitatively by injecting the fuel through the plate arrangement shown in Fig. 1b. In one set of the experiments with methane the slit behind the injection port hole was closed (full squares in Fig. 2b). In this case the ignition conditions are worse than for transverse nozzle injection (dash-dotted curve in Fig. 2b). Most probably this result is due to the fact that entrainment of ambient air into the wake downstream of the fuel jet is prevented by the plate. Hence, there recirculates a mixture that is richer in methane and has higher ignition temperatures and larger ignition delay times than for nozzle injection.

Therefore, it was to be expected that the ignition conditions for the plate could be improved by introducing air into the wake, thereby shifting the methane air ratio towards leaner values. In fact, experiments in which the slit was open, yielded essentially lower ignition temperatures: for methane the ignition temperature dropped by $\approx$300°C (hollow squares in Fig. 2b). For hydrogen (hollow circles) the ignition temperature was lower than for nozzle injection at $\alpha = 90°$ (dash-dotted curve) over the whole range of pressure ratios. As the static air stream temperature could not be reduced below 500°C, the minimum ignition temperature for hydrogen was estimated to be about 470°C by extrapolating the two branches of the curve.

These experiments show that the ignition conditions can be improved considerably when the fuel gas is injected through a hole in a wall, provided that entrainment of ambient air into the wake downstream of the injection port hole is made possible. Even lower ignition temperatures should be reached by injecting the fuel through a boring in the wall which is inclined upstream to the air flow.

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Swirling Nozzle Flow Equations from Crocco's Relation

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Introduction

Swirling nozzle flow is not simple to analyze because it varies radially as well as axially. For example, controversies[1–5] on the axial velocity at the choking throat have not yet been settled because the exact flow equations were not used. However, Lewellen et al.[6] have determined the exact governing equations for flow varying only in the radial direction, i.e., in a constant area region. The purpose of this Note is to show that the swirling phenomena of an inviscid nonisoenergetic and nonisentropic flow are essentially governed by Crocco's relation. It is shown here that Lewellen's result can be obtained directly from this relation. Solutions for some simple swirling flows are given below.

Crocco's Equation

For a steady flow Crocco's equation relates the variation of entropy, vorticity, and total enthalpy in the following manner:

$$T\nabla S + \mathbf{V} \times \mathbf{\Omega} = \nabla h_0 \tag{1}$$

where T,S, $\mathbf{V}$,$\mathbf{\Omega}$ and h_0 denote, respectively, the temperature, entropy, velocity, vorticity, and total enthalpy of the fluid. For an axisymmetrical flow it is convenient to use cylindrical coordinates (r,θ,z). In this case the radial variation of Crocco's equation appears as

$$T\, \partial S/\partial r + v\Omega_z + w\, \partial w/\partial r = \partial h_0/\partial r \tag{2}$$

where v and w represent, respectively, the tangential and axial velocity components. The axial derivative of radial velocity was neglected since the nozzle cross-sectional area was assumed to vary slowly. For an axisymmetrical flow the vorticity component in the axial direction may be related to the

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